Ability of engineering undergraduates to solve real function limit problems

Capacidad de los estudiantes de ingeniería para resolver problemas de límite de funciones reales

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Recibido 13 de septiembre de 2021, aceptado 03 de diciembre de 2022
Received: September 13, 2021 Accepted: December 03, 2022

ABSTRACT

The society of the 21st century requires training of engineers capable of solving problems in applications of the limit of real functions. In this paper, the mathematical skills of three engineering degrees belonging to the 2019 cohort of a public university is analyzed. The objectives of this study are to investigate the skills of undergraduate students in solving of mathematical problems, defined according to their nature in routine and non-routine, and according to their context, in real, realistic, fantasist and purely mathematical. Also, identify and characterize the types of problems addressed, and to solicit the opinion about the level of difficulty of the problems involving the limit of real functions and on the prior knowledge that they need to achieve a better performance in their resolution. Students from three different engineering programs volunteered to participate in the study. The evaluation tool used consisted of a function limit application problem solving test, which was content validated by eight experts in the field, and an open-ended opinion questionnaire. The results favor the performance of the students in solving routine problems in fantasist contexts, and even are guarantee development the mathematical skills necessary to solve non-routine problems of application of limits.

Keywords: Problem solving, types of mathematical problems, limits, engineering programs.

RESUMEN

La sociedad del siglo XXI requiere la formación de ingenieros capaces de resolver problemas en aplicaciones del límite de funciones reales. En este trabajo se analizan las habilidades matemáticas de tres ingenierías pertenecientes a la cohorte 2019 de una universidad pública. Los objetivos de este estudio son investigar las habilidades de los estudiantes de pregrado en la resolución de problemas matemáticos, definidos según su naturaleza en rutinarios y no rutinarios, y según su contexto, en real, realista, fantasista y puramente matemático. Además, identificar y caracterizar los tipos de problemas abordados y solicitar la opinión sobre el nivel de dificultad de los problemas que involucran el límite de funciones reales y sobre el conocimiento previo que necesitan para lograr un mejor desempeño en su resolución. Los estudiantes de tres programas de ingeniería diferentes se ofrecieron como voluntarios para participar en el estudio. La herramienta de evaluación utilizada consistió en una prueba de resolución de tipos de problemas de aplicación de límite de función real, la cual fue validada por contenido por ocho expertos en la materia, y un cuestionario de opinión abierto. Los resultados favorecen el desempeño de los estudiantes en la resolución de problemas rutinarios en contextos fantasistas e incluso garantizan el desarrollo de habilidades matemáticas necesarias para resolver problemas no rutinarios de aplicación de límites.

Palabras clave: Resolución de problemas, tipos de problemas matemáticos, límites, carreras de ingeniería.

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INTRODUCTION

In academic contexts in the area of mathematics, it has been shown that there is a general disarticulation between the calculation of the limit and the understanding of its meaning due to its complexity and abstraction. Likewise, it must be added that correctly solving problems related to the calculation of the limit of a function does not guarantee its understanding. In many cases, students are taught to solve problems related to the calculation of limits in a more or less mechanical way [1-2]. Therefore, they need guidance to accomplish them and to reach a satisfactory understanding of the concept.

Problem-solving will make engineering students see the need to strengthen their knowledge to face increasingly complex challenges. Solving problems at any level of mathematics or technical subjects requires creative skills that often need to emerge. It is imperative to correct the structuring of their knowledge at the conceptual, reflective, and practical levels [3-4].

Understanding the concept of limits is fundamental in differential calculus university classes [5]. The mathematical object limit is the first significant obstacle for engineering students in learning higher-level university mathematics. Researchers have found that students have three main difficulties in understanding limits: the infinite processes of limits, the formal definition of limits, and the value of limits [6-14]. Findings of [8-9, 11-12], and [15] indicate that students use incorrect metaphorical reasoning to understand limits, which finally turns into difficulties in solving both exercises and application problems. The research results of reference [16] suggest that, at the end of their mathematics courses, many engineering students do not consider using a formal definition of limit to solve mathematical problems based on limits. Despite its importance, it is difficult for students to understand the concept [17-18].

At present, there is concern about the proper training of engineers. In Europe, we can mention an important work from the United Kingdom [19]. The authors of the work reported in reference [20] developed and presented the need to work in the global dimension of the engineer, which highlights the importance of developing specific disciplinary skills as well as those related to areas of science, technology, engineering, and mathematics, called STEM areas [20-23] and other so-called generic skills, such as holistic thinking [24]. Specific disciplinary competences of the engineer, abstraction and reasoning capacity, communication and mathematical argumentation, simplification and mathematization of a problem, verification and validation of a result, and others are considered. They are recognized as STEM skills, the resolution of complex problems, critical thinking, teamwork, communication, reasoning and analysis skills, concentration, creativity and innovation, and generation of ideas, among others.

It is important to emphasize that these skills also appear in documents in Latin America as part of the competencies that the 21st-century citizen is expected to possess, specifically the future engineer [25-26]. However, the authors in reference [26] emphasize in their conclusions that students are often disinterested or unmotivated in learning mathematics since they do not observe relationships between the concepts, they learn in mathematics class and real-world applications. Within this context, we surveyed undergraduate engineering programs whose competency-based curriculum includes problem-solving.

The research questions were the following: Do engineering students of the mathematical skills necessary to solve problems of limited applications of real variable functions? What types of problems present the most and the least difficulties to solve?

A study was designed for three engineering careers whose general objective was to identify and characterize the types of problems by applying a mathematics test. Moreover, to know the opinion and performance of engineering students at a state university in problems solving of application limits fundamental functions, the following specific objectives were formulated: (1) to elaborate, to validate, and to apply an open response test to solve actual function boundary problem types, (2) to know the degree of easiness and difficulty of the problems, (3) to know and characterize the missing knowledge and previous concepts necessary for solving the problem types.

THEORETICAL FRAMEWORK

The authors [27] detailed engineering skills needs conducted in countries around the world, such as the

United States, Australia, the United Kingdom, and Malaysia. The synthesis of skills and competencies of more than 200 studies that were carried out allowed classifying the global skills into four main dimensions. Dimension II, referring to Mental Cognition and Thinking, includes problem-solving competence [28]. In this dimension, skills are required for current and future engineering graduates. One of them is the resolution of problems whose generic definition extracted from the different references and studies is: use knowledge systematically to identify, analyze, formulate, solve, and evaluate complex and multidisciplinary problems - by applying cognitive skills (logical, critical, and creative thinking).

Because of the importance of problem-solving skills, it has been the subject of research in higher education. Reference [29] shows how university students present learning difficulties associated with the logical operations of thinking according to the demands of the current level of teaching; also, they use strategies learned by heart to solve problems that are often complex, a method that is not very effective for reasoning and obtaining an adequate result [4].

However, a standard textbook introduces the concept of a limit when defining Riemann integrals (and then again with improper integrals) and, of course, with sequences and series. Nevertheless, limits are never covered in any comprehensive, detailed way; instead, they are frequently introduced to move on to differentiation and then used when needed to introduce various other components of single or multivariable calculus, mostly using exercises but not problems. Along the same lines, [30] indicate that the activities carried out in the classroom influence how students understand and solve problems; added to this is the importance of registers of semiotic representation in the teaching of calculus, whose lack of various types of registers becomes an epistemological obstacle.

For [31], quoted in [3], a problem can also be considered as a proposition that is formulated from specific known data to find the numerical value or result corresponding to the question suggested; in turn [31], refers to problems as situations that are genuinely problematic and also it does not exist a routine procedure for resolution; it is also considered to be situations that do not only prepare students to solve current problems but also form and develop particularities that allow them to solve creatively.

Types of mathematical problems

For this study, as a theoretical framework, the types of mathematical problems that the authors [32-34] have been working on in different areas of mathematics were taken. According to this classification, problems are considered according to their nature and context. According to their nature, problems are classified into routine and non-routine ones, and according to their context, they are classified as accurate, realistic, fantasist, and purely mathematical (Figure 1).

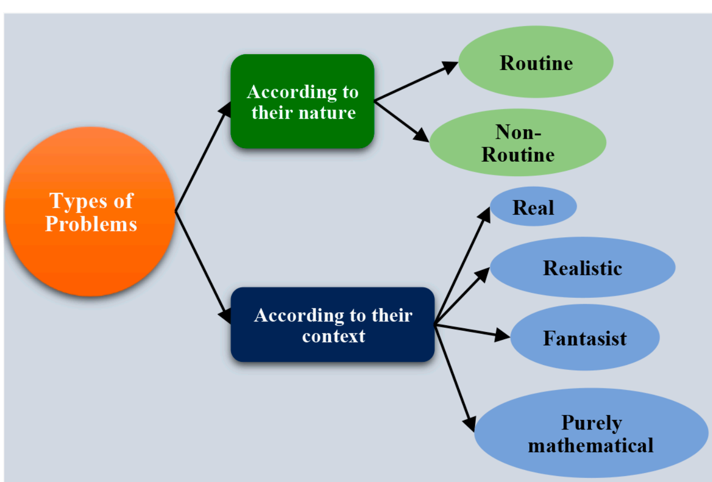


Figure 1. Types of mathematical problems.

Nature of the problem

Based on their nature, problems are defined as Routine and Non-Routine ones. Routine problems are like those solved during instructional courses; the student follows a sequence that involves understanding the concepts and algorithms to achieve valid solutions.

A problem will be ***Non-Routine*** when a student does not know an answer, a previously established procedure, or routine to find it.
Example test problem Nº 4: "*A culture of bacteria in the solar system 2000 light years from Earth, grows according to the law:* $y = \dfrac{1,25}{1+0,25e^{-0,4t}}$ *where time t ≥ 0 is measured in hours and the weight of the culture in grams.*

a) *Determine the weight of the culture after 60 minutes.*
b) *What will be the weight of it when the number of hours grows indefinitely?*"

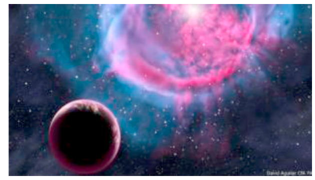
Problem Context

- **Real context problem**: A context is real if it is produced in reality and compromises the actions of the student in it.
  Example: "*Stand in front of a door. You decide to open it, so you move closer. You realize you are getting closer and closer, but you can't touch the door handle. You try to get closer and closer, but there is always space between your hand and the door handle, no matter how hard you try. That action you take has the mathematical name of limit"*.
- **Realistic context problem**: A context is realistic if it is susceptible to being produced and it is about a simulation of reality or a part of it.
  Example test problem Nº 2: "*The amount of a drug in the bloodstream t hours after it is injected intramuscularly is given by the function* $f(t) = \dfrac{10t}{t^2+1}$.

Over time, what is the limit amount of drug in the blood?"

- **Fantasist context problem**: A context is a fantasist if it is a figmet of the imagination and is unfounded in reality.
  Example test problem Nº 3: "*The pressure on the seabed of the most recently discovered planet Kepler 438b is given according to the function:*

  $$P = \frac{5h - h^2 + 3h^3}{2h + h^3 + h^4}$$

  where h is the density. What value does the pressure approach when the density approaches 0?"
- **Purely mathematical context**: A context is purely mathematical if it makes exclusive reference to mathematical objects: numbers, relations, arithmetic operations, geometric and figures.
  Example test problem Nº 1: *¿How close to 2 should x be taken so that 8x-5 is at a distance 11 less than 0.001?*

METHODOLOGY

The research was quantitative; in particular, a descriptive study [35] was conducted with a transactional design, and the variables were measured in a single session [36-37]. The study sample of the investigation corresponded to 40 volunteer students of intact courses that, during the second semester of 2019, took the compulsory course of Calculus I and belonging to the undergraduate programs: Engineering 1, Engineering 2, and Engineering 3 of the Universidad de Los Lagos in Chile. The instruments and evaluation techniques used were a mathematics test, and an opinion questionnaire.

Mathematics test

The initial assessment instrument consisted of a test to solve eleven open-response problems applying limits of functions based on the classification of types of mathematical problems according to their nature and context. This test was previously validated for content by the judgment of eight experts in the field [38]. An 85% congruence between their answers was considered accepted as valid, the type of problem that was finally considered in the test [39]. The final test was left with eight problems (Table 1) and was applied in mid-December for 120 minutes, and by the end of the Calculus I course; therefore, they had already been evaluated in the limited content. Any intervention by the collaborators who applied the evaluation instruments was avoided so as not to influence the process.

Table 1. Distribution of types of mathematical problems.

Problems	Types of problems routines and non-routines
Problem 1	Routine and purely mathematical context .
Problem 2	Routine and realistic context.
Problem 3	Routine and fantasist context.
Problem 4	Non-routine.
Problem 5	Routine and realistic context.
Problem 6	Routine and purely mathematical context.
Problem 7	Non-routine.
Problem 8	Routine and fantasist context.

Table 2. Problem-solving grading scale.

Score	Solution stages
0	Level 0 corresponds to the absence of the content that describes the problem.
1	Level 1 corresponds to a partial performance response, indicating comprehension of the problem, yet faces difficulties easily.
2	Level 2 corresponds to a partial performance response to the resolution of problem, yet few mistakes lead to a wrong final solution.
3	Level 3 corresponds answer where it has been used proper method used and it has reached to the correct solution.

The internal consistency of the test was evaluated using Cronbach's alpha statistic, with which a value of 0.89 was obtained. An adaptation of the scale of [40] was used to assess the level of achievement in the test, which assigns 0 to 3 points per problem according to the student's level of progress in solving it.

The Levene test showed that the variances across groups are homoscedastic, that is, all three samples come from a population with the same variance (sig < 0,005). The degrees of internal discrimination were established through the biserial correlation (rpbis), which was, on average, equal to 0.38. Overall, the difficulty of each problem and the test was 37.5%.

Opinion questionnaire

Once the mathematics test was completed, the students were given an open-ended opinion questionnaire, which gave an account of the process lived regarding the abilities and difficulties of the problems, previous concepts, and missing knowledge that they considered necessary in their opinion to adequately solve the problems, which was answered anonymously and voluntarily by all the students.

RESULTS

General test analysis

Figure 2 presents the level of achievement of the students of the three engineering programs in the problem-type resolution test, evaluated according to the scale of [40].

In general, the students of Engineering 1 revealed a better performance in the problems of limit applications, achieving 71%. As for the results by

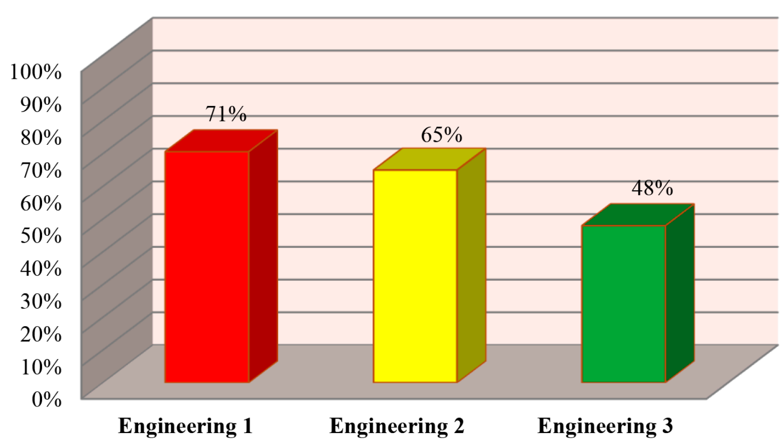


Figure 2. Level of achievement in solving types of problems.

type of problem, Figure 3 shows the performance percentage of the three engineerings.

According to Figure 3, the three highest percentages of achievement were reached in the resolution of routine problem number 3 of fantasist context (66%) and problems number 2 and 8 of realistic context with 56.6% and of fantasist context with 54.6%, respectively, which means that most used an adequate procedure and managed to reach the correct solution of the problem, achieving level 3 of the rating scale.

Example test problem Nº 8 of fantasist context: "*The bank offers the credit card "Master Plop". Through data obtained in the past, they have determined that the percentage of collection of the ones given in one month is in function of the time after granting them. This function.*

$$P = f(r) = 0.76\left(1 - 2^{-0.05r}\right)$$

P is accounts receivable "r" percentage months after granting the card. Which percentage is expected to be collected after 2 and 5 months? If the number of months elapsed from the "Master Plop Card" granting time growths indefinitely, determine the percentage expected to be collected".

The three lowest percentages correspond to problem number 5 of realistic context (16.8%) and the remaining two to problem number 1 (40%) and number 6 (29.7%), both routine of purely mathematical context. According to the scale, levels 2 and 3 clearly understand a problem. When analyzing the scores obtained by the students in these last two levels, the levels of achievement in both routine problems of fantasist context and realistic context are maintained. However, non-routine problems numbers 7 and 4 also stand out, whose achievement scores, adding levels 2 and 3 of the rating scale, reach 79.9% and 60.4%, respectively.

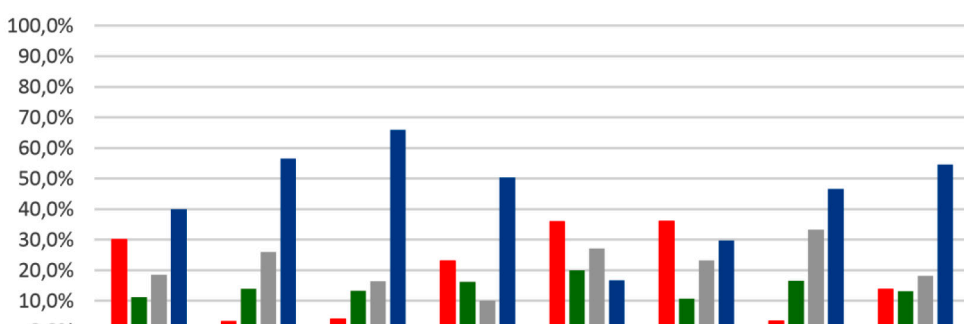
The following is an engineering student's solution to a routine problem in a purely mathematical context.

Example test problem Nº 6: " Study if there exists the $\lim_{x \to 1} f(x)$ where

$$f(x) = \begin{cases} \dfrac{2 - \sqrt{x+3}}{x-1}, & si\ x > 1 \\ \dfrac{2x^2 - 3}{x^2 + 3}, & si\ x < 1 \end{cases}$$

Is it possible to define f (1) so that f is continuous at x = 1?

A context is purely mathematical if it makes exclusive reference toexcludes mathematical objects: numbers, relations, arithmetic operations, geometric figures, and others. Basically, it consists



	Problem 1	Problem 2	Problem 3	Problem 4	Problem 5	Problem 6	Problem 7	Problem 8
0	30,3%	3,4%	4,3%	23,3%	36,0%	36,2%	3,5%	14,0%
1	11,2%	14,0%	13,3%	16,3%	20,0%	10,8%	16,6%	13,2%
2	18,5%	26,0%	16,4%	10,0%	27,2%	23,3%	33,3%	18,2%
3	40,0%	56,6%	66,0%	50,4%	16,8%	29,7%	46,6%	54,6%

Figure 3. Percentage of performance on each type of problem.

of problems with calculation and definitions of the most common type that appear in the conventional evaluation of mathematics. In general, one of the problems presented the most significant difficulty in achieving the correct resolution (Figure 4).

In answer to the problem, errors due to non-logically valid inferences, errors due to theorems, or distorted definitions can be observed. The student incorrectly applied the lateral limits theorem, evaluating both functions' limits to the left and right. Furthermore, he needs help understanding the definition of continuity.

Questionnaire analysis

Next, the general results of the opinion questionnaire with four open-ended questions are presented (Figure 5) and applied to all the students of the three engineering programs. First, they were surveyed regarding the problems they solved, considering their degree of difficulty and their arguments in this matter. They were also inquired about the previous concepts that students considered when answering the problem-solving test on limits of real functions and, finally, the knowledge they perceived lacking to achieve satisfactory performance.

Figure 5 shows in detail that the problems that, according to the student's perception, caused them the least difficulties were problems 2, 3, and 4, with response percentages of 84.3, 78.3, and 63.7, respectively. In turn, these problems correspond to routine problems of realistic context and fantasist (2 and 3) and non-routine (4). Problems 5, 6 and 1, with values equal to 45.7%, 45.2%, and 43.4%, were considered extremely difficult by all the students.

In Figure 6, it can be seen that 59% of the students indicate that in order to be able to solve the test, it was necessary to know limits and continuity; in addition, 50% of the students indicated that, as a previous concept, needed to have reading comprehension. The functions and those derived also recognize them as previous concepts to solve limit applications problems of a real variable function. It is understood that some students concluded that they could use the knowledge of derivatives to solve some problems; however, even though they needed to remember the definition of limits correctly, they decided to derive.

It can be seen in Figure 7 that 75% of the students indicated that they lacked study and they also needed to have greater clarity of the concept of limit (32.1%), which is the central content of the test. Remarkably, only 11.8% recognize how their lack of knowledge in solving limit application problems is precisely general problem-solving. Example of

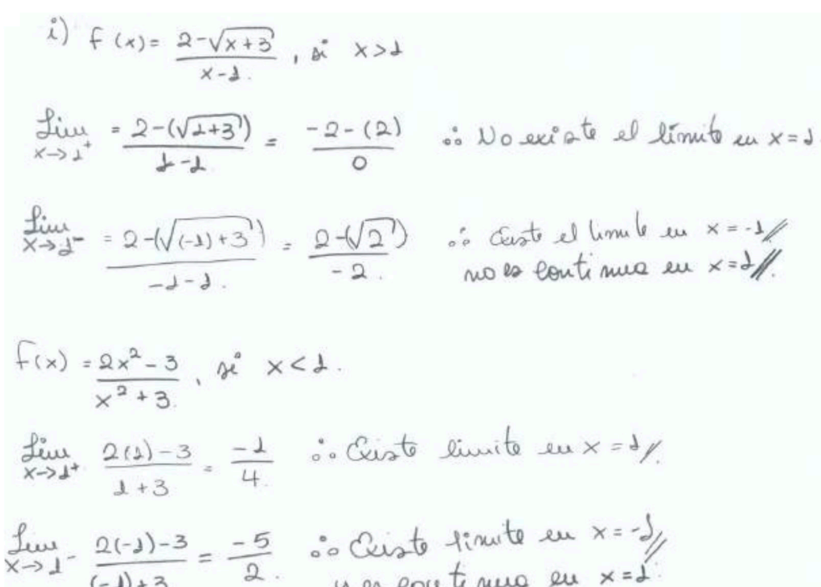


Figure 4. Answer by a student to problem.

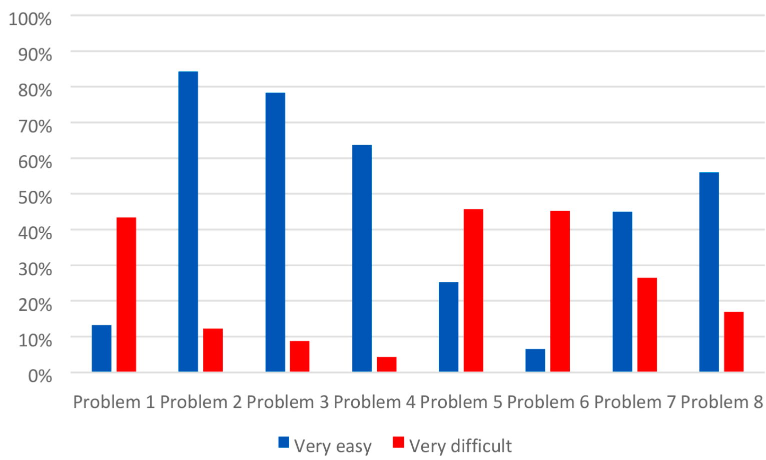


Figure 5. Degree of difficulty in each type of problem.

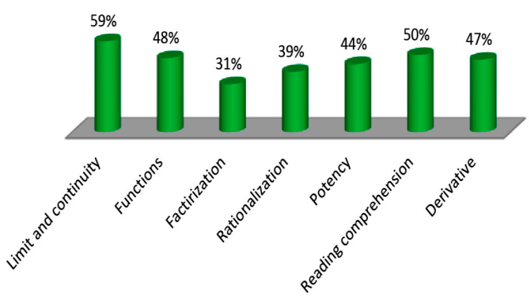


Figure 6. Preliminary concepts.

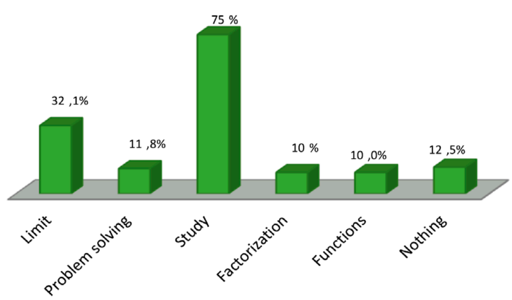
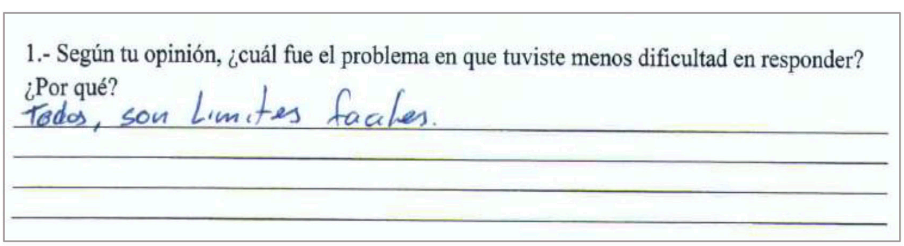


Figure 7. Missing knowledge.


1.- Según tu opinión, ¿cuál fue el problema en que tuviste menos dificultad en responder? ¿Por qué?

Todos, son Limites faciles.

Figure 8. Answer to student to question 1 of the opinion questionnaire.

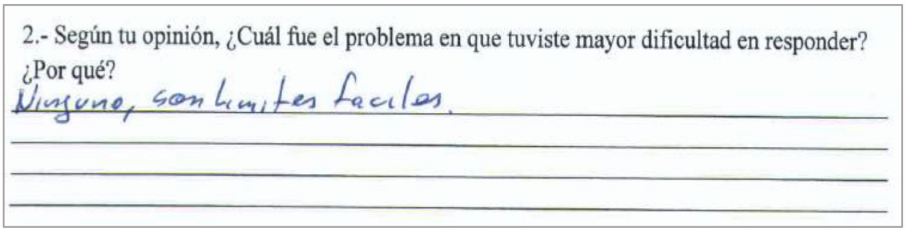
a student's opinions about the degree of difficulty of the problems:

1. In your opinion, what was the problem in which you had the less difficulty in answering? *All of them are easy limits.* Figure 8.
2. In your opinion, what was the problem in which you had the greater difficulty in answering? *One of the concepts was lateral limits.*

The student's responses reflect total confidence in their knowledge and effective resolution of contextualized problems.

Example of a student's opinions regarding the use of prior knowledge. Figure 9.

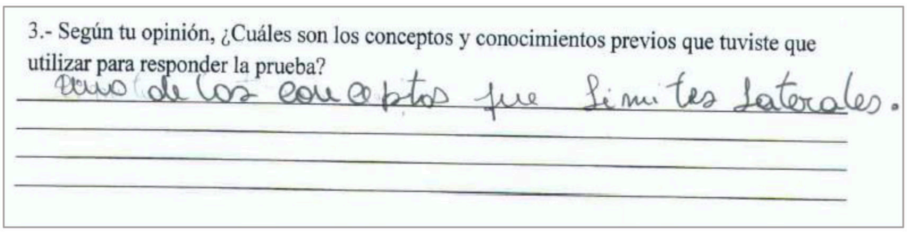
3. In your opinion, what previous knowledge or concepts did you need to answer the test? *I think I missed reviewing and studying.* Figure 10.



2.- Según tu opinión, ¿Cuál fue el problema en que tuviste mayor dificultad en responder? ¿Por qué?

Ninguno, son limites faciles.

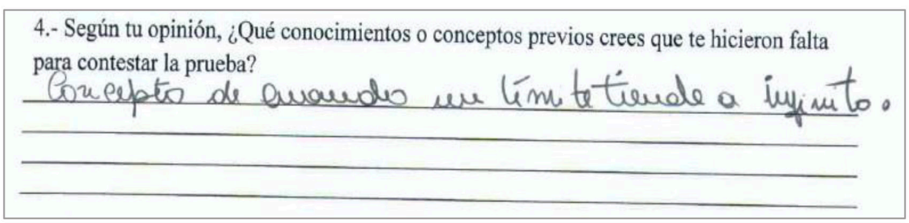
Figure 9. Answer to student to question 2 of the opinion questionnaire.



3.- Según tu opinión, ¿Cuáles son los conceptos y conocimientos previos que tuviste que utilizar para responder la prueba?

Uno de los conceptos fue limites laterales.

Figure 10. Answer to student to question 3 of the opinion questionnaire.



4.- Según tu opinión, ¿Qué conocimientos o conceptos previos crees que te hicieron falta para contestar la prueba?

Conceptos de cuando un limite tiende a infinito.

Figure 11. Student's answer to question 4 of the opinion questionnaire.

4. In your opinion, what previous knowledge or concepts did you need to answer the test? *I think I missed reviewing and studying.* Figure 11.

The student does not give arguments to justify why he had greater or lesser difficulty responding to problems in general. Neither gave her opinion in question 3 regarding the concepts and prior knowledge required to answer in its entireness the mathematics test.

CONCLUSIONS

Problem-solving is of paramount importance in engineering education, and the results obtained reflect a good performance of the students in general in solving application problems where the limit should be used. From the assessment of non-routine and routine contextual problem-solving performance in three engineering programs, it is necessary to include types of problems that allow contextualizing one mathematical content considered by the literature of high complexity in its understanding, in this case, the limit of real functions.

Two routine problems with a fantasy context were incorporated into this study to show the situations that mathematics can assist in solving. It is necessary to highlight the performance of the students in these problems, that is to say, in those that are figments of the imagination, therefore, they do not have a foundation in reality, and practically they are not found in the literature of the teaching of limit; however, they were trendy among the students, because in addition to being different, they attracted their attention, motivated the student, facilitated the understanding of concepts attaining higher levels of resolution thereof.

Two non-routine problems were also incorporated into the study. A problem will be considered non-routine when a student does not know an answer or a predetermined procedure to find it. It should be mentioned that performance remained the same as the difficulty level of the problem considered non-routine increased.

The context of problems can change from experiences that are familiar to students to applications involved with other disciplines. The idea is that the problems cover relevant mathematical concepts of the curriculum and, by choosing the right level and familiarity with the students, improvements in mathematical learning can be achieved, which, later on, will be the support to tackle and solve more complex or non-routine problems.

Finally, the main contribution of this study has been to provide types of mathematical problems that attract the student to their resolution, also to provide a better understanding of the deficiencies that students bring from high school in mathematics and that greatly influence engineering calculation courses, specifically in the appropriation of the concept of limit of a function, as well as providing a different idea of how instructors can use different types of problems in their calculus courses.

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